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QUASI-STATIC TRANSIENT THERMAL STRESSES IN AN INFINITE WEDGE*

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Abstract—The plane transient thermoelastic problem for an infinite wedge subjected to an instantaneous heat source at an arbitrary location is considered. The problem is formulated in terms of displacements and Mellin–Laplace transforms are used for the solution. A real integral representation of the stress field is given. The main emphasis is placed on the analysis of the singular behavior of stresses around the apex for large values of the wedge angle, 2β. It is found that in the symmetric case for $\beta > \pi/2$ and in the antisymmetric case for $\beta > 0{\cdot}715\pi$ the apex is a point of singularity for the stresses. The power of this singularity as well as the variation of the stresses in θ and that of the stress intensity factor in time is studied and some numerical results are given.

NOTATION

A_1, A_2	stress intensity factors
E, ν, λ, μ	elastic constants
k	coefficient of heat conduction
$k^* =$	$\alpha(3\lambda+2\mu)$
$k_1 =$	$(2n+1)\pi/2\beta$
$k_2 =$	$(n+1)\pi/\beta$
p	Mellin parameter
q_0	intensity of the heat source
r, θ, z, t	space and time coordinates
r', θ'	location of the heat source
s	Laplace parameter
T	temperature
u, v	r, θ-components of displacement vector
α	coefficient of thermal expansion
β	half-wedge angle
κ	thermal diffusivity
ϕ_j, ψ_j	poles in p-plane
$\omega =$	$\dfrac{E\alpha q_0}{\beta(1-\nu)}$ (in figures)

1. INTRODUCTION

EXISTING solutions of transient thermoelastic problems are, for the most part, relatively recent in origin. Among the notable solutions we may mention those given by Melan [1] for quenching of a uniformly heated sphere, Sternberg [2] for the infinite medium with a spherical cavity, Bailey [3] for the half-space subjected to a sudden change in temperature on a circular portion of its plane boundary, Jaunzemis and Sternberg [4] for the semi-infinite slab subjected to a sudden change in temperature on a finite segment of its edge, and Youngdahl and Sternberg [5] for an infinitely long elastic circular shaft subjected to a sudden change in surface temperature along a finite band. Under steady temperature fields, a certain degeneracy in the stresses exists if the medium is simply-connected.

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For example, the body is stress-free if the problem is two-dimensional [6, 7] and, in a three-dimensional problem for the half-space under given thermal boundary conditions on its plane surface, the stress field is plane and parallel to the boundary [8, 9].

In multiply-connected domains under steady temperature fields, even though the solutions are few, the existence of thermal stresses are well known [e.g. 6, 10]. In the case of infinite media subjected to one-dimensional uniform heat flow disturbed by the existence of insulated holes or cracks, thermal stresses develop in the neighborhood of cavities and have the same singular character as those induced by mechanical disturbances [11–14].

In the literature, there appears to be a lack of information concerning the singular behavior of thermal stresses in media containing sharp notches and undergoing transient temperature fields. Hence, the primary objective of this paper will be the study of thermal stresses, particularly their behavior in the vicinity of the points of singularity, in a configuration simple enough to lend itself to tractable analysis and general enough to have some practical applications. The infinite plane wedge of an arbitrary angle under z-independent temperature field is selected to be such a configuration. The thermal stress problem of a plane wedge under steady temperature fields was studied by Piechocki and Zorski [15] and by Piechocki [16]. In [15] a plane wedge of an arbitrary angle, clamped along the straight edges and subjected to time-independent temperature boundary conditions is considered. The solution is given in terms of Mellin inversion integrals and contains no discussion of the behavior of stresses—singular or otherwise. [16] considers a plane wedge with stress-free boundaries subjected to a steady-state, z-independent heat source at an arbitrary point in x–y plane. The straight edges of the wedge are held at zero temperature. The results include the stresses in a quarter plane given in terms of improper integrals and those for a half plane which agree with the expressions found in [7]. However the limitation of the solution obtained in [16] is that it applies only to those wedges with angles less than π; hence, the heat source is the only point of singularity for stresses.

In what follows we consider a wedge of an arbitrary angle with stress-free boundaries which is subjected to a heat pulse at an arbitrary location. The problem may either be one of plane strain, in which case the source strength is assumed to be independent of z, or it may be one of generalized plane stress, where it is assumed that the plane boundaries, $z = \mp h/2$, are thermally insulated, h being the thickness. Since the main objective of the paper is to provide the necessary Green's functions to be used in the analysis of wedges with time-varying heat generation (or surface heating, in the case of thin plates) and since the time rates in such cases are usually small compared to velocities of stress waves, the present analysis is restricted to the quasi-static case, that is, the inertia effects are ignored. Recent studies on dynamic thermoelasticity seem to bear out the validity of this assumption [5, 10, 17, 18]. Strictly speaking, because of the stress-free bounding surfaces of the wedge, in the neighborhood of the apex, the behavior of dynamic stresses may be quite different to those obtained on the basis of quasi-static assumption. However, the difference will be in the time-dependent stress intensity factors rather than the strength of the stress singularities. Moreover, this difference may be significant only if the time rate of change of temperature boundary conditions or thermal loading is extremely high.* Hence, if one may judge on the basis of available though limited

* For example, a ramp duration of less than 10^{-12} sec in ramp-type heating of a semi-infinite steel medium [17].

numerical work [17, 18], under physically realistic conditions, the error in stresses and displacements which results from ignoring the inertia effects may be assumed to be negligible.

Largely for mathematical expediency, also ignored are the effect of temperature on the thermoelastic constants along with the thermoelastic coupling and the anelastic behavior of the material.

To avoid the solution of another boundary-value problem, a formulation directly in terms of displacements rather than a displacement potential is used.

2. FORMULATION OF THE PROBLEM

In a quasi-static plane strain problem with z-independent temperature and stress fields, the heat conduction and the equilibrium equations may be written as

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{q}{k}=\frac{1}{\kappa}\frac{\partial T}{\partial t} \tag{1}$$

$$\begin{aligned}(\lambda+2\mu)\frac{\partial e}{\partial r}-\frac{2\mu}{r}\frac{\partial \omega_z}{\partial \theta}-(3\lambda+2\mu)\alpha\frac{\partial T}{\partial r}&=0\\ (\lambda+2\mu)\frac{1}{r}\frac{\partial e}{\partial \theta}+2\mu\frac{\partial \omega_z}{\partial r}-(3\lambda+2\mu)\frac{\alpha}{r}\frac{\partial T}{\partial \theta}&=0\end{aligned} \tag{2}$$

where k is the coefficient of heat conduction, q is the heat generated in unit volume and unit time, κ is the coefficient of diffusivity. λ and μ are Lamé's constants, α is the coefficient of linear thermal expansion, T is the temperature and the dilatation e and the rotation ω_z are given in terms of r and θ components of the displacements u and v as follows:

$$e=\frac{\partial(ru)}{r\partial r}+\frac{\partial v}{r\partial \theta},\qquad \omega_z=\frac{1}{2r}\left(\frac{\partial(rv)}{\partial r}-\frac{\partial \mu}{\partial \theta}\right). \tag{3}$$

Consider now an infinite wedge, $r>0$, $-\beta\le\theta\le\beta$, with zero initial temperature and stresses and subjected to an instantaneous heat source at $t=0$, $r=r'$ and $\theta=\theta'$. To obtain the stress distribution (1) and (2) will have to be solved subject to the following initial and boundary conditions:

$$\begin{aligned}&T(r,\theta,t)=0, \qquad t\le 0\\ &T(r,\theta,t)=0, \qquad \theta=\pm\beta, \qquad t>0\\ &q(r,\theta,t)=q_0\frac{1}{r}\delta(r-r')\delta(\theta-\theta')\delta(t-t').\end{aligned} \tag{4}$$

$$\begin{aligned}&\sigma_{ij}(r,\theta,t)=0, \qquad t\le 0, \qquad (i,j=r,\theta)\\ &\sigma_{\theta\theta}(r,\theta,t)=0, \qquad \sigma_{r\theta}(r,\theta,t)=0, \qquad \theta=\pm\beta, t>0\end{aligned} \tag{5}$$

where q_0 is the intensity of the heat source (BTU per unit thickness).

The solution of (1) subject to (4), with the wedge angle taken as $0\le\theta\le\theta_0$, was obtained by Carslaw and Jaeger [19], which, after performing the integration [20, p. 395], may be written as

$$T(r,\theta,t) = \frac{q_0}{\theta_0}\sum_{n=1}^{\infty}\frac{1}{\kappa t}\exp\left(-\frac{r^2+r'^2}{4\kappa t}\right)I_s\left(\frac{rr'}{2\kappa t}\right)\sin s\theta\sin s\theta'$$
$$s = \frac{\pi n}{\theta_0} \tag{6}$$

Expressing (6) in the wedge $-\beta \le \theta \le \beta$ and dividing it into symmetric and anti-symmetric components, T_1 and T_2, corresponding to heat sources $q_0/2$ at $r = r'$, $\theta = \pm\theta'$ and $q_0/2$ at $r = r'$, $\theta = \theta'$ and $-q_0/2$ at $r = r'$, $\theta = -\theta'$, respectively, we obtain

$$T_1 = \frac{q_0}{2\kappa\beta t}\exp\left(-\frac{r^2+r'^2}{4\kappa t}\right)\sum_{n=0}^{\infty}I_{k_1}\left(\frac{rr'}{2\kappa t}\right)\cos k_1\theta'\cos k_1\theta,$$
$$k_1 = (2n+1)\pi/2\beta. \tag{7}$$

$$T_2 = \frac{q_0}{2\kappa\beta t}\exp\left(-\frac{r^2+r'^2}{4\kappa t}\right)\sum_{n=0}^{\infty}I_{k_2}\left(\frac{rr'}{2\kappa t}\right)\sin k_2\theta'\sin k_2\theta,$$
$$k_2 = (n+1)\pi/\beta. \tag{8}$$

To solve (2) we will use successively Mellin–Laplace transformations in variables r and t. For a given suitably well-behaved function $f(r, t)$ the Mellin–Laplace transform pairs are formally defined as

$$\hat{\bar{f}}(p,s) = \int_0^{\infty} e^{-st}\,dt\int_0^{\infty} f(r,t)r^{p-1}\,dr$$
$$f(r,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} r^{-p}\,dp\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\hat{\bar{f}}(p,s)e^{st}\,ds. \tag{9}$$

Taking into account the initial conditions, assuming that within the strip of regularity containing the Bromwitch line of the Mellin inversion integral the functions T, u and v are such that

$$r^pT,\ r^{p-1}u,\ r^p\frac{\partial u}{\partial r},\ r^{p-1}\frac{\partial u}{\partial\theta},\ r^{p-1}v,\ r^p\frac{\partial v}{\partial r},\ r^{p-1}\frac{\partial v}{\partial\theta} \to 0$$

as

$$r \to 0 \quad \text{and} \quad r \to \infty \tag{10}$$

and integrating by parts, from (2) and (3) we obtain

$$\frac{d^2\hat{\bar{u}}}{d\theta^2}+B_1\frac{d\hat{\bar{v}}}{d\theta}+B_2\hat{\bar{u}}+B_3\hat{\bar{T}} = 0$$
$$\frac{d^2\hat{\bar{v}}}{d\theta^2}+B_4\frac{d\hat{\bar{u}}}{d\theta}+B_5\hat{\bar{v}}-B_6\frac{d\hat{\bar{T}}}{d\theta} = 0 \tag{11}$$

where $\hat{\bar{T}}$, $\hat{\bar{u}}$, $\hat{\bar{v}}$ are, respectively, the Mellin–Laplace transforms of T, $r^{-1}u$ and $r^{-1}v$ and the constants are given by

$$B_1 = -2-\frac{A}{\mu}p+p, \qquad B_2 = \frac{A}{\mu}p(p-2), \qquad B_3 = \frac{k^*p}{\mu}$$
$$B_4 = 2+\frac{\mu}{A}p-p, \qquad B_5 = \frac{\mu}{A}p(p-2), \qquad B_6 = \frac{k^*}{A} \tag{12}$$
$$A = \lambda+2\mu, \qquad k^* = (3\lambda+2\mu)\alpha.$$

Using the stress displacement relations of thermoelasticity in cylindrical coordinates and integrating by parts, the Mellin–Laplace transforms of stresses may be obtained as

$$\hat{\bar{\sigma}}_{rr} = \lambda\left(\hat{\bar{u}}+\frac{\mathrm{d}\hat{\bar{v}}}{\mathrm{d}\theta}\right)-A(p-1)\hat{\bar{u}}-k^*\hat{\bar{T}}$$
$$\hat{\bar{\sigma}}_{\theta\theta} = A\left(\hat{\bar{u}}+\frac{\mathrm{d}\hat{\bar{v}}}{\mathrm{d}\theta}\right)-\lambda(p-1)\hat{\bar{u}}-k^*\hat{\bar{T}} \tag{13}$$
$$\hat{\bar{\sigma}}_{r\theta} = \mu\left(\frac{\mathrm{d}\hat{\bar{u}}}{\mathrm{d}\theta}-p\hat{\bar{v}}\right).$$

Thus the differential equations (11) will be subject to following boundary conditions:

$$A\left(\hat{\bar{u}}+\frac{\mathrm{d}\hat{\bar{v}}}{\mathrm{d}\theta}\right)-\lambda(p-1)\hat{\bar{u}}-k^*\hat{\bar{T}} = 0, \qquad \theta = \pm\beta$$
$$\frac{\mathrm{d}\hat{\bar{u}}}{\mathrm{d}\theta}-p\hat{\bar{v}} = 0, \qquad \theta = \pm\beta. \tag{14}$$

Finally defining

$$\hat{\bar{F}}_j(p,s) = \int_0^\infty e^{-st}\,\mathrm{d}t\int_0^\infty \frac{1}{2\kappa t}\exp\left(-\frac{r^2+r'^2}{4\kappa t}\right)I_j\left(\frac{rr'}{2\kappa t}\right)r^{p-1}\,\mathrm{d}r. \tag{15}$$

The Mellin–Laplace transforms of symmetric and anti-symmetric temperature fields, (7), become

$$\hat{\bar{T}}_1 = \frac{q_0}{\beta}\sum_{n=0}^{\infty}\hat{\bar{F}}_{k_1}\cos k_1\theta'\cos k_1\theta, \qquad k_1 = (2n+1)\pi/2\beta$$
$$\hat{\bar{T}}_2 = \frac{q_0}{\beta}\sum_{n=0}^{\infty}\hat{\bar{F}}_{k_2}\sin k_2\theta'\sin k_2\theta, \qquad k_2 = (n+1)\pi/\beta. \tag{16}$$

Throughout the paper the interchange of summation and integration is assumed to be permissible. Although complete rigor would require its justification at each step, this is inferred with knowledge that the summation occurs through the assumed temperature distribution, which is uniformly convergent for all values of time except at $r = r'$, $\theta = \theta'$ for $t = 0$.

3. SOLUTION IN TERMS OF INFINITE INTEGRALS

First consider the symmetric case. Substituting $\hat{\bar{T}} = \hat{\bar{T}}_1$ from (16) and using the boundary conditions, (14), after some algebra, the solution of (11) may be obtained as

$$\hat{u} = \frac{q_0 k^*}{\beta A} \sum_{n=0}^{\infty} \left(\frac{(p-2)\hat{\bar{F}}_{k_1} \cos k_1\theta' \cos k_1\theta}{k_1^2-(p-2)^2} + \frac{(-1)^n k_1 \hat{\bar{F}}_{k_1} \cos k_1\theta'}{[k_1^2-(p-2)^2]G(p,\beta)} \left[(p-2)\cos p\beta \cos(p-2)\theta - \left(p+\frac{2\mu}{\lambda+\mu}\right)\cos(p-2)\beta \cos p\theta \right] \right)$$

$$\hat{v} = \frac{q_0 k^*}{\beta A} \sum_{n=0}^{\infty} \left(\frac{k_1 \hat{\bar{F}}_{k_1} \cos k_1\theta' \sin k_1\theta}{k_1^2-(p-2)^2} + \frac{(-1)^n k_1 \hat{\bar{F}}_{k_1} \cos k_1\theta'}{[k_1^2-(p-2)^2]G(p,\beta)} \left[(p-2)\cos p\beta \sin(p-2)\theta - \left(p-\frac{2A}{\lambda+\mu}\right)\cos(p-2)\beta \sin p\theta \right] \right),$$

$$k_1 = (2n+1)\pi/2\beta,$$

$$G(p,\beta) = (p-1)\sin 2\beta + \sin 2(p-1)\beta. \tag{17}$$

Observing that in (17) Laplace parameter s appears only through $\hat{\bar{F}}_{k_1}(p,s)$, Laplace inversions of (17), i.e., the Mellin transforms of displacements, u, v, are obtained by simply replacing $\hat{\bar{F}}_{k_1}$ by $\bar{F}_{k_1}$, which is given by

$$\bar{F}_{k_1}(p,t) = \int_0^{\infty} \frac{1}{2\kappa t} \exp\left(-\frac{r^2+r'^2}{4\kappa t}\right) I_{k_1}\left(\frac{rr'}{2\kappa t}\right) r^{p-1}\, dr. \tag{18}$$

Using (13) the Mellin transforms of the stresses may then be written as

$$\bar{\sigma}_{rr} = \frac{q_0 E\alpha}{\beta(1-\nu)} \sum_{n=0}^{\infty} \frac{\bar{F}_{k_1} \cos k_1\theta'}{G(p,\beta)[k_1^2-(p-2)^2]} [(-1)^n k_1(p-1)(p+2) . \cos(p-2)\beta \cos p\theta - (-1)^n k_1(p-1)(p-2)\cos p\beta \cos(p-2)\theta - G(p,\beta)(k_1^2+p-2)\cos k_1\theta],$$

$$\bar{\sigma}_{\theta\theta} = \frac{q_0 E\alpha}{\beta(1-\nu)} \sum_{0}^{\infty} \frac{\bar{F}_{k_1} \cos k_1\theta'}{G(p,\beta)[k_1^2-(p-2)^2]} [(-1)^n k_1(p-1)(p-2) . \cos p\beta \cos(p-2)\theta - (-1)^n k_1(p-1)(p-2)\cos(p-2)\beta \cos p\theta + G(p,\beta)(p-1)(p-2)\cos k_1\theta], \tag{19}$$

$$\bar{\sigma}_{r\theta} = \frac{q_0 E\alpha}{\beta(1-\nu)} \sum_0^\infty \frac{\bar{F}_{k_1} \cos k_1\theta'}{G(p,\beta)[k_1^2-(p-2)^2]} [(-1)^n k_1 p(p-1) .$$

$$. \cos(p-2)\beta \sin p\theta - (-1)^n k_1(p-1)(p-2) \cos p\beta \sin(p-2)\theta$$

$$- G(p,\beta) k_1(p-1) \sin k_1\theta],$$

$$k_1 = (2n+1)\pi/2\beta.$$

The stresses are obtained by using the Mellin inversion theorem:

$$\sigma_{ij}(r,\theta,t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \bar{\sigma}_{ij}(p,\theta,t) r^{-p} \, dp, \qquad (i,j = r,\theta). \tag{20}$$

Equation (19) indicates that, since the Mellin inversion of $\bar{F}_{k_1}$ is known, by writing

$$\bar{\sigma}_{ij}(p,\theta,t) = \sum_{n=0}^{\infty} \bar{F}_{k_1}(p,t) \bar{\tau}_{ijn}(p,\theta), \qquad (i,j = r,\theta) \tag{21}$$

the stresses may be obtained as infinite series of convolution integrals:

$$\sigma_{ij}(r,\theta,t) = \sum_{n=0}^{\infty} \int_0^\infty F_{k_1}\left(\frac{r}{x}, t\right) \tau_{ijn}(x,\theta) \frac{dx}{x} \tag{22}$$

where

$$F_{k_1}(r,t) = \frac{1}{2\kappa t} \exp\left(-\frac{r^2+r'^2}{4\kappa t}\right) I_{k_1}\left(\frac{rr'}{2\kappa t}\right) \tag{23}$$

and τ_{ijn} $(i,j = r,\theta;\ n = 0,1,2,\ldots)$ are the inversions of $\bar{\tau}_{ijn}$ which are the coefficients of $\bar{F}_{k_1}$ in (19).

The solutions σ_{ij} or τ_{ijn} are not affected by the particular choice of c in inversion if the line of integration in p plane is varied within the same strip of regularity common to all integrands. The appropriate strip of regularity in turn is dictated by the conditions (10). From a close examination of $\bar{\sigma}_{rr}$, $\bar{\sigma}_{\theta\theta}$ and $\bar{\sigma}_{r\theta}$ it can be shown that within the strip $\frac{1}{2} < Re(p) < \frac{3}{2}$ the functions are regular and a choice of c in this strip, say $c = 1$, complies with conditions (10).

To evaluate τ_{ijn} one may use the residue theorem by completing the contour in the half planes $Re(p) > 1$ for $r > 1$ and $Re(p) < 1$ for $r < 1$. The only difficulty in following this procedure lies in the evaluation of the poles of $\bar{\tau}_{ijn}(p)$.

Taking the line of integration at $c = 1$ and writing $p = 1+iy$, τ_{ijn} may also be expressed in terms of real integrals as follows:

$$\tau_{rrn}(r,\theta) = \frac{q_0 E\alpha \cos k_1\theta'}{\pi\beta(1-\nu)} \int_0^\infty \Big\{(-1)^n k_1 \frac{M_1}{N_1 N_2} [(2k_1^2 y - 2y + 2y^3) .$$

$$. \cos(y \log r) + 4y^2 \sin(y \log r)] - (-1)^n k_1 \frac{M_2}{N_1 N_2} .$$

$$. [(y^4 - 3y^2 + k_1^2 y^2) \cos(y \log r) - (3y^3 + k_1^2 y - y) \sin(y \log r)]$$

$$- \frac{\cos k_1\theta}{N_2} [(k_1^4 - 2k_1^2 + k_1^2 y^2 + y^2 + 1) \cos(y \log r)$$

$$+ (y^3 + y - yk_1^2) \sin(y \log r)]\Big\} \frac{dy}{r}$$

$$\tau_{\theta\theta n}(r,\theta) = \frac{q_0 E\alpha\cos k_1\theta'}{\pi\beta(1-\nu)}\int_0^\infty \Big\{(-1)^n k_1 \frac{M_2}{N_1 N_2}[(y^4+y^2+k_1^2y^2)\,. \\ .\cos(y\log r)+(k_1^2y-y-y^3)\sin(y\log r)] \\ -\frac{\cos k_1\theta}{N_2}[(y^4+y^2+k_1^2y^2)\cos(y\log r) \\ -(y^3+y-k_1^2y)\sin(y\log r)]\Big\}\frac{dy}{r} \tag{24}$$

$$\tau_{r\theta n}(r,\theta) = \frac{q_0 E\alpha\cos k_1\theta'}{\pi\beta(1-\nu)}\int_0^\infty \Big\{(-1)^n k_1 \frac{M_3}{N_1 N_2}[2y^3\cos(y\log r) \\ +(y^4+k_1^2y^2-y^2)\sin(y\log r)]+(-1)^n k_1\frac{M_4}{N_1N_2}\,. \\ .\,[2y^2\cos(y\log r)+(y^3+k_1^2y-y)\sin(y\log r)]-\frac{k_1\sin k_1\theta}{N_2}\,. \\ .\,[2y^2\cos(y\log r)+(k_1^2y-y+y^3)\sin(y\log r)]\Big\}\frac{dy}{r}$$

$$\begin{aligned} M_1 &= \cos(\beta-\theta)\cosh(\beta+\theta)y+\cos(\beta+\theta)\cosh(\beta-\theta)y \\ M_2 &= \sin(\beta-\theta)\sinh(\beta+\theta)y+\sin(\beta+\theta)\sinh(\beta-\theta)y \\ M_3 &= \sin(\beta+\theta)\cosh(\beta-\theta)y-\sin(\beta-\theta)\cosh(\beta+\theta)y \\ M_4 &= \cos(\beta-\theta)\sinh(\beta+\theta)y-\cos(\beta+\theta)\sinh(\beta-\theta)y \\ N_1 &= y\sin 2\beta+\sinh 2\beta y \\ N_2 &= [y^2+(k_1+1)^2][y^2+(k_1-1)^2] \\ k_1 &= (2n+1)\pi/2\beta. \end{aligned}$$

Integrals of the form of (24) appear in literature in connection with other problems in elasticity and are found to lend themselves to numerical treatment [e.g., 21–23]. Some of the integrals of (24) can also be evaluated in closed form by using contour integration [24].

In a similar way, the Mellin transforms of the stresses for the anti-symmetric case may be obtained as follows:

$$\bar{\sigma}_{rr} = \frac{q_0 E\alpha}{\beta(1-\nu)}\sum_{n=0}^{\infty}\frac{\bar{F}_{k_2}\sin k_2\theta'}{H(p,\beta)[k_2^2-(p-2)^2]}[(-1)^n k_2(p-1)(p+2)\,. \\ .\sin(p-2)\beta\sin p\theta-(-1)^n k_2(p-1)(p-2)\sin p\beta\sin(p-2)\theta \\ -H(p,\beta)(k_2^2+p-2)\sin k_2\theta],$$

$$\bar{\sigma}_{\theta\theta} = \frac{q_0 E\alpha}{\beta(1-\nu)} \sum_{n=0}^{\infty} \frac{\bar{F}_{k2} \sin k_2\theta'}{H(p,\beta)[k_2^2-(p-2)^2]}[(-1)^n k_2(p-1)(p-2) . \\ . \sin p\beta \sin(p-2)\theta - (-1)^n k_2(p-1)(p-2)\sin(p-2)\beta . \\ . \sin p\theta + H(p,\beta)(p-1)(p-2)\sin k_2\theta],$$

$$\bar{\sigma}_{r\theta} = \frac{q_0 E\alpha}{\beta(1-\nu)} \sum_{n=0}^{\infty} \frac{\bar{F}_{k_2} \sin k_2\theta'}{H(p,\beta)[k_2^2-(p-2)^2]}[(-1)^n k_2(p-1)(p-2) . \\ . \sin p\beta \cos(p-2)\theta - (-1)^n k_2 p(p-1)\sin(p-2)\beta \cos p\theta \\ + H(p,\beta)(p-1)k_2 \cos k_2\theta],$$

$$H(p,\beta) = (p-1)\sin 2\beta - \sin 2(p-1)\beta, \tag{25}$$

$$\bar{F}_{k_2}(p,t) = \frac{1}{2\kappa t}\int_0^\infty \exp\left(-\frac{r^2+r'^2}{4\kappa t}\right) I_{k_2}\left(\frac{rr'}{2\kappa t}\right) r^{p-1}\,dr,$$

$$k_2 = (n+1)\pi/\beta.$$

Again, it can be shown that $\frac{1}{2} < Re(p) < \frac{3}{2}$ is the appropriate strip of regularity and $c = 1$ may be chosen for the Mellin inversion integrals. Noting the similarity of (19) and (25), the stresses for this case may also be expressed in terms of infinite integrals similar to (22).

4. ASYMPTOTIC BEHAVIOR OF STRESSES FOR SMALL r

For $r < 1$, (19) and (25) may be inverted through the use of the residue theorem by completing the contour in the half plane $Re(p) < 1$. If the behavior of stresses in the vicinity of the apex of the wedge is of primary interest, we need to examine merely the asymptotic behavior of these inversions for small values of r. The leading terms in the asymptotic expansion of the stresses will be contributed by the residues at those poles of $\bar{\sigma}_{ij}$ which lie closest to the line $Re(p) = c = 1$.

It can be shown that at the zeros of $k_1^2-(p-2)^2$ the integrands in both, symmetric and anti-symmetric cases, have no singularities except for $\beta = \pi$, $n = 1$, or $p = \frac{1}{2}$, which is only a simple pole. Hence, the remaining singularities will occur at zeros of $G(p, \beta)$ and $H(p, \beta)$ for the symmetric and anti-symmetric cases, respectively, and may be obtained from

$$G = (p-1)\sin 2\beta + \sin 2(p-1)\beta = 0, \qquad p = \phi_1, \phi_2, \ldots \\ H = (p-1)\sin 2\beta - \sin 2(p-1)\beta = 0, \qquad p = \psi_1, \psi_2, \ldots. \tag{26}$$

The first zeros ϕ_1 and ψ_1 of G and H are shown in Fig. 1 as a function of β. ϕ_1 and ψ_1 are the only singularities with positive real parts. As will be seen below, since the stresses are of the form $r^{-\phi_k}$, $r^{-\psi_k}$, the remaining poles and the continuation of ϕ_1, ψ_1 into the negative range have no significance in the treatment of the singular behavior of the stresses, and hence are not displayed in Fig. 1. We note that $p = \frac{1}{2}$ is the pole of $\bar{\sigma}_{ij}$ lying closest to $p = c = 1$, ϕ_1 and ψ_1 are either real or appear in complex conjugate pairs, according to (26) they are symmetrically located with respect to $p = 1$ and hence in the strip $\frac{1}{2} < Re(p) < \frac{3}{2}$ the functions $\bar{\sigma}_{ij}(p)$ are regular $(i, j = r, \theta)$, as previously stated.

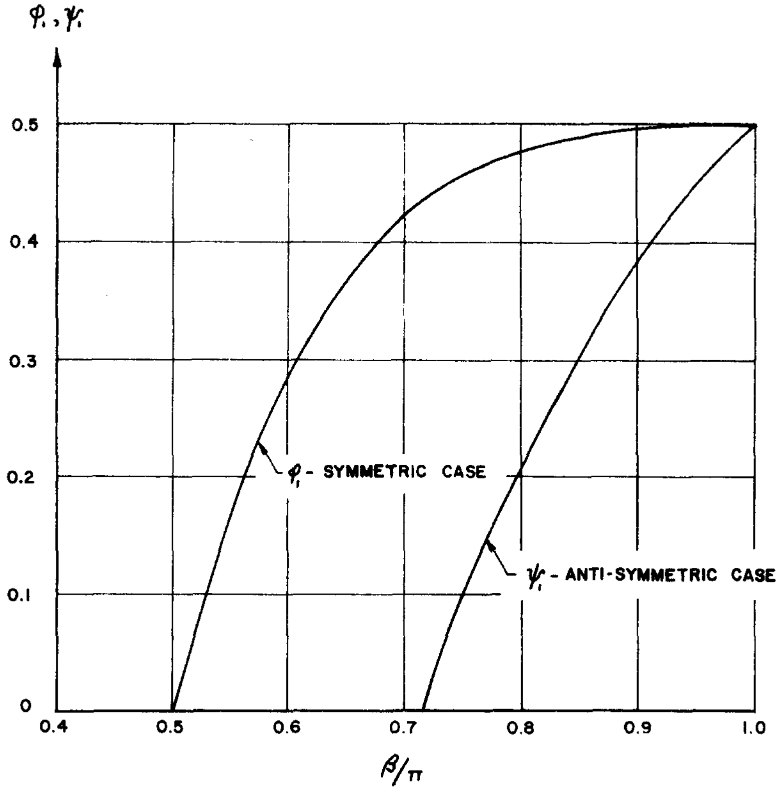

FIG. 1. The zeros ϕ_1 of $G(p, \beta)$ and ψ_1 of $H(p, \beta)$ as a function of β.

In the symmetric case, for $\beta = \pi$ the leading term will be contributed by the pole $p = \frac{1}{2}$ for $n = 1$, the functions $\bar{\sigma}_{ij}(p)$ have no singularities at the zeros of $G(p)$, and hence the residue theorem gives

$$\tau_{rr_1}(r, \theta) = \frac{E\alpha q_0}{4\pi(1-\nu)} r^{-\frac{1}{2}} \cos\frac{\theta}{2}\left(1+\sin^2\frac{\theta}{2}\right)$$

$$\tau_{\theta\theta_1}(r, \theta) = \frac{E\alpha q_0}{4\pi(1-\nu)} r^{-\frac{1}{2}} \cos^3\frac{\theta}{2} \tag{27}$$

$$\tau_{r\theta_1}(r, \theta) = \frac{E\alpha q_0}{8\pi(1-\nu)} r^{-\frac{1}{2}} \sin\theta \cos\frac{\theta}{2}.$$

Thus, from (22) we obtain

$$
\begin{aligned}
\sigma_{rr}(r,\theta,t) &= \frac{E\alpha q_0}{8\pi(1-\nu)\kappa t} r^{-\frac{1}{2}} \cos\left(\frac{3\theta'}{2}\right) \cos\frac{\theta}{2}\left(1+\sin^2\frac{\theta}{2}\right) f(t) + 0(r^0) \\
\sigma_{\theta\theta}(r,\theta,t) &= \frac{E\alpha q_0}{8\pi(1-\nu)\kappa t} r^{-\frac{1}{2}} \cos\left(\frac{3\theta'}{2}\right) \cos^3\frac{\theta}{2} f(t) + 0(r^0) \\
\sigma_{r\theta}(r,\theta,t) &= \frac{E\alpha q_0}{16\pi(1-\nu)\kappa t} r^{-\frac{1}{2}} \cos\left(\frac{3\theta'}{2}\right) \sin\theta \cos\frac{\theta}{2} f(t) + 0(r^0) \\
f(t) &= \frac{2\kappa t}{r'^{\frac{3}{2}}\Gamma(\frac{3}{2})} \gamma\left(\frac{3}{2}, \frac{r'^2}{4\kappa t}\right)
\end{aligned}
\tag{28}
$$

where $\gamma(a, x)$ is the incomplete gamma function.

For $\beta < \pi$ the leading terms in the asymptotic expansions of stresses will be contributed by the residue at the pole $p = \phi_1$ and from the residue theorem it follows that

$$
\begin{aligned}
\tau_{ijn}(r,\theta) &= \tau^*_{ijn}(r,\theta) + 0(r^{-\phi_2}), \qquad (i, j = r, \theta) \\
\tau^*_{rrn}(r,\theta) &= \frac{E\alpha q_0 r^{-\phi_1}}{\beta(1-\nu)g_n}(-1)^n k_1(\phi_1 - 1)[(\phi_1+2)\cos(\phi_1-2)\beta\cos\phi_1\theta \\
&\qquad -(\phi_1-2)\cos\phi_1\beta\cos(\phi_1-2)\theta] \\
\tau^*_{\theta\theta n}(r,\theta) &= \frac{E\alpha q_0 r^{-\phi_1}}{\beta(1-\nu)g_n}(-1)^n k_1(\phi_1-1)(\phi_1-2)[\cos\phi_1\beta\cos(\phi_1-2)\theta \\
&\qquad -\cos(\phi_1-2)\beta\cos\phi_1\theta] \\
\tau^*_{r\theta n}(r,\theta) &= \frac{E\alpha q_0 r^{-\phi_1}}{\beta(1-\nu)g_n}(-1)^n k_1(\phi_1-1)[\phi_1\cos(\phi_1-2)\beta\sin\phi_1\theta \\
&\qquad -(\phi_1-2)\cos\phi_1\beta\sin(\phi_1-2)\theta] \\
g_n &= [k_1^2 - (\phi_1-2)^2][\sin 2\beta + 2\beta\cos 2(\phi_1-1)\beta] \\
k_1 &= (2n+1)\pi/2\beta.
\end{aligned}
\tag{29}
$$

Thus, the convolution integrals, (22), give the stresses as

$$
\begin{aligned}
\sigma_{rr} &= \sum_{n=0}^{\infty} \frac{1}{2\kappa t} \exp\left(-\frac{r'^2 t}{4\kappa t}\right) \cos k_1\theta' \tau^*_{rrn} f_n(t) + 0(r^{-\phi_2}) \\
\sigma_{\theta\theta} &= \sum_{n=0}^{\infty} \frac{1}{2\kappa t} \exp\left(-\frac{r'^2}{4\kappa t}\right) \cos k_1\theta' \tau^*_{\theta\theta n} f_n(t) + 0(r^{-\phi_2}) \\
\sigma_{r\theta} &= \sum_{n=0}^{\infty} \frac{1}{2\kappa t} \exp\left(-\frac{r'^2}{4\kappa t}\right) \cos k_1\theta' \tau^*_{r\theta n} f_n(t) + 0(r^{-\phi_2}) \\
f_n(t) &= \frac{r'^{k_1}(4\kappa t)^{(\phi_1-k_1)/2}\Gamma(\phi_1/2 + k_1/2)}{2\Gamma(k_1+1)} {}_1F_1\left(\frac{\phi_1}{2} + \frac{k_1}{2}; k_1+1; \frac{r'^2}{4\kappa t}\right)
\end{aligned}
\tag{30}
$$

where ${}_1F_1(a;c;z)$ is the confluent hypergeometric series.

Similarly, in the anti-symmetric case for $r < 1$, $\beta \leq \pi$, the stresses may be expressed as

$$\sigma_{rr} = \frac{E\alpha q_0 r^{-\psi_1}}{\beta(1-\nu)}(\psi_1-1)[(\psi_1+2)\sin(\psi_1-2)\beta\sin\psi_1\theta$$
$$-(\psi_1-2)\sin\psi_1\beta\sin(\psi_1-2)\theta]h(t)+0(r^{-\psi_2})$$

$$\sigma_{\theta\theta} = \frac{E\alpha q_0 r^{-\psi_1}}{\beta(1-\nu)}(\psi_1-1)(\psi_1-2)[\sin\psi_1\beta\sin(\psi_1-2)\theta$$
$$-\sin(\psi_1-2)\beta\sin\psi_1\theta]h(t)+0(r^{-\psi_2}) \tag{31}$$

$$\sigma_{r\theta} = \frac{E\alpha q_0 r^{-\psi_1}}{\beta(1-\nu)}(\psi_1-1)[(\psi_1-2)\sin\psi_1\beta\cos(\psi_1-2)\theta$$
$$-\psi_1\sin(\psi_1-2)\beta\cos\psi_1\theta]h(t)+0(r^{-\psi_2})$$

$$h(t) = (4\kappa t)^{(\psi_1-2)/2}\exp\left(-\frac{r'^2}{4\kappa t}\right)\sum_{n=0}^{\infty}\left[(-1)^n\left(\frac{r'^2}{4\kappa t}\right)^{k_2/2}\Gamma\left(\frac{\psi_1}{2}+\frac{k_2}{2}\right)\sin k_2\theta'\,.\right.$$
$$\left.\cdot\,{}_1F_1\left(\frac{k_2}{2}+\frac{\psi_1}{2};k_2+1;\frac{r'^2}{4\kappa t}\right)\right]\Big/\{\Gamma(k_2)[k_2^2-(\psi_1-2)^2][\sin 2\beta-2\beta\cos 2(\psi_1-1)\beta]\}$$

$$k_2 = (n+1)\pi/\beta.$$

5. NUMERICAL RESULTS AND DISCUSSION

The comparison of the results given by (30) and (31) with those given in [25] and [26] indicates that the form and the power of the stress singularities (i.e. $r^{-\phi_1}$, $r^{-\psi_1}$) in plane wedges are the same for mechanical and transient thermal loadings. For the infinite plate with a semi-infinite cut and subjected to an instantaneous heat source, q_0, at $t = 0$, $r = r'$, $\theta = \theta'$, combining (28) and (31), the stress state in the neighborhood of the end of the cut is obtained as

$$\sigma_{rr} = \frac{1}{r^{\frac{1}{2}}}\cos\frac{\theta}{2}\left[A_1(t)\left(1+\sin^2\frac{\theta}{2}\right)+A_2(t)\left(\frac{3}{2}\sin\theta-2\tan\frac{\theta}{2}\right)\right]+0(r^0)$$

$$\sigma_{\theta\theta} = \frac{1}{r^{\frac{1}{2}}}\cos\frac{\theta}{2}\left[A_1(t)\cos^2\frac{\theta}{2}-\frac{3}{2}A_2(t)\sin\theta\right]+0(r^0) \tag{32}$$

$$\sigma_{r\theta} = \frac{1}{2r^{\frac{1}{2}}}\cos\frac{\theta}{2}[A_1(t)\sin\theta+A_2(t)(3\cos\theta-1)]+0(r^0)$$

where the stress intensity factors are given by

$$A_1(t) = \frac{E\alpha q_0 \cos\frac{3\theta'}{2}\gamma\left(\frac{3}{2},\frac{r'^2}{4\kappa t}\right)}{2(1-\nu)(\pi r')^{\frac{3}{2}}}$$

$$A_2(t) = \frac{E\alpha q_0}{2\pi(1-\nu)}\sum_{n=0}^{\infty}\frac{(-1)^n\left(\frac{r'^2}{4\kappa t}\right)^{(n+1)/2}\Gamma\left(\frac{2n+3}{4}\right)\sin(n+1)\theta'{}_1F_1\left(\frac{2n+3}{4};n+2;\frac{r'^2}{4\kappa t}\right)}{(4\kappa t)^{\frac{3}{4}}\exp\left(\frac{r'^2}{4\kappa t}\right)n!(n-\frac{1}{2})(n+\frac{5}{2})} \tag{33}$$

In its r, θ dependence, the stress field given by (32) is identical to the solution of an equivalent elastic body under static loads (e.g. [27]).

For other wedge angles, the stress field around the apex may be expressed as

$$\sigma_{ij} = A_1(t,\beta)r^{-\phi_1}B_{ij}(\theta)+A_2(t,\beta)r^{-\psi_1}C_{ij}(\theta)+O(r^{-\phi_2}, r^{-\psi_2})$$

where (the real parts of) ϕ_2 and ψ_2, that is, the powers of the singularity vary with β in accordance with Fig. 1 which indicates that for $\beta \leq \pi/2$ in the symmetric case and for $\beta \leq 0{\cdot}715\pi$ in the anti-symmetric case, the stresses at $r = 0$ will be bounded. The curves shown in Fig. 1 are identical to those found in [26] for the symmetric case and [25] for the anti-symmetric case. However, the locations of the leading poles in the anti-symmetric case for all values of β and $p < 0$ are at the zeros of $H(p)$ and do not experience a distinct change at $\beta = 0{\cdot}715\pi$ as found in [25]. The significance of the angle $\beta = 0{\cdot}715\pi$ was pointed out by Sternberg and Koiter in [25]. It was found that $0{\cdot}715\pi$ is the largest half-wedge angle for which the classical solution for a wedge subjected to a concentrated couple at the apex is valid. This type of breakdown of the theory is due to the fact that for wedges with larger angles under concentrated thermal or mechanical loads, there are two points of singularities—the apex of the wedge, $r = 0$, and the application point of the load, $r = r'$, $\theta = \theta'$. As r' approaches zero, the stress intensity factors A_1 and A_2 go to infinity and the solution becomes meaningless.*

Figures 2–8 show some of the numerical results. Figures 2–6 show the variation in relative magnitude of the stress intensity factors as a function of β and the θ-dependence of the stresses in the vicinity of the apex for small values of time. The direction for maximum $\sigma_{\theta\theta}$ remains to be $\theta = 0$ as wedge angle varies, however for maximum σ_{rr} the angle goes from $\theta = 70{\cdot}5°$ to $\theta = 90°$ as β goes from π to $\pi/2$. Figure 7 shows the variation of $\sigma_{\theta\theta}r^{\phi_1}$ at $\theta = 0$ or, essentially, the stress intensity factor as a function of β and t.† Figure 8 shows the stress intensity factor in the case of $\beta = \pi$ as a function of time.

Finally, from Fig. 1, we observe that in a plane wedge with a half-angle β, $0{\cdot}715\pi < \beta < \pi$, the strength of the stress singularity for the symmetric loading is always greater than that for the anti-symmetric loading and hence, from the view point of practical applications, the symmetric stress state is by far the more critical one. Partly for this reason, most of the numerical results given by the figures refer to the symmetric case.

* Note, for example, the problem of a cracked plate subjected to wedge loadings on the crack surface where application points of the load approach the crack-tip. In this case, $A_1 \sim (r-r')^{-\frac{1}{2}}$ and $A_1 \to \infty$ as $r \to r'$.

† r' appearing on the abcissa of Figs. 7 and 8 should be considered as a constant in studying the figures. Since they appear elsewhere in the expressions of A_1 and A_2 the figures do not reflect the actual dependence of A_1 and A_2 on r' (see, for example, equation (33)).

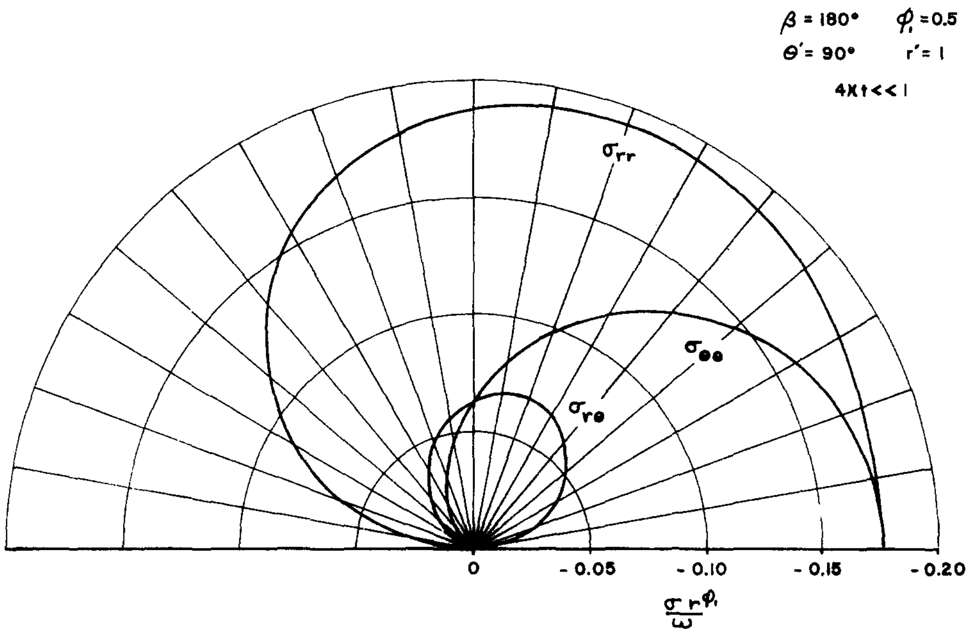

FIG. 2. Stresses near the apex of a wedge ($\beta = 180°$)—symmetric temperature distribution.

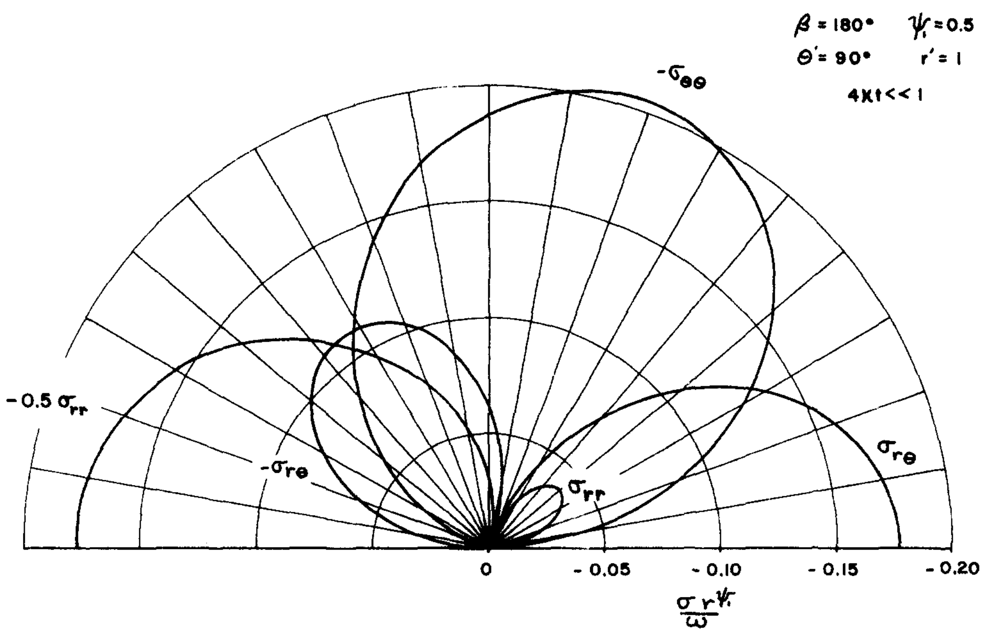

FIG. 3. Stresses near the apex of a wedge ($\beta = 180°$)—anti-symmetric temperature distribution.

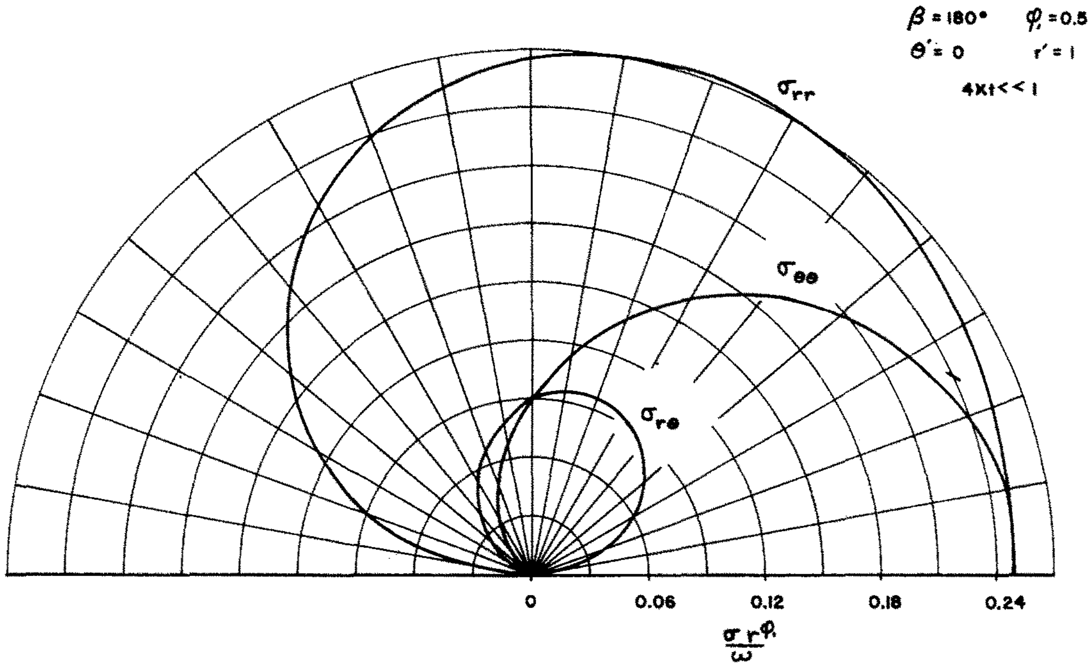

FIG. 4. Stresses near the apex of a wedge ($\beta = 180°$)—symmetric temperature distribution.

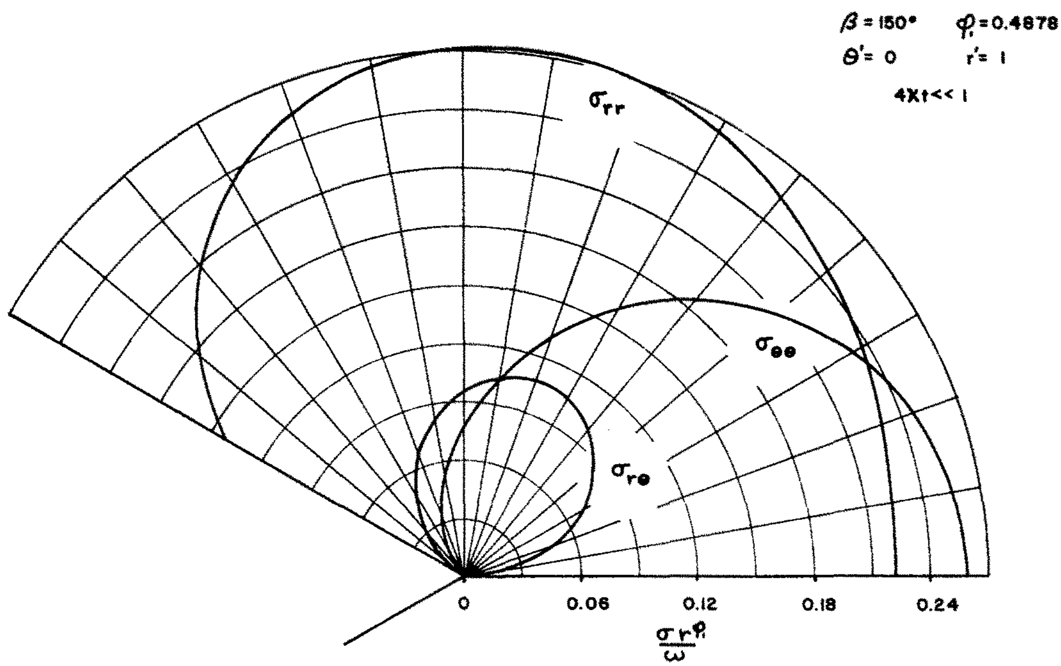

FIG. 5. Stresses near the apex of a wedge ($\beta = 150°$)—symmetric temperature distribution.

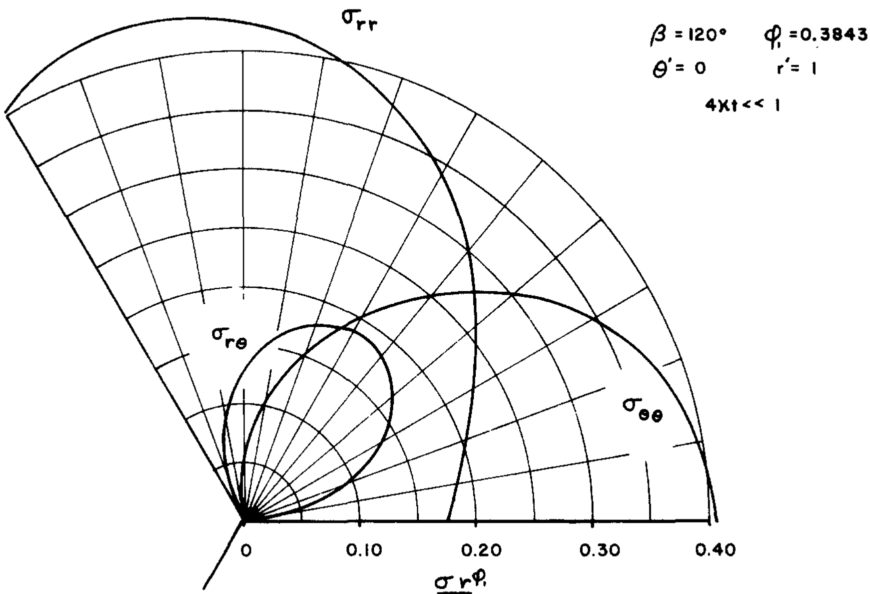

FIG. 6. Stresses near the apex of a wedge ($\beta = 120°$)—symmetric temperature distribution.

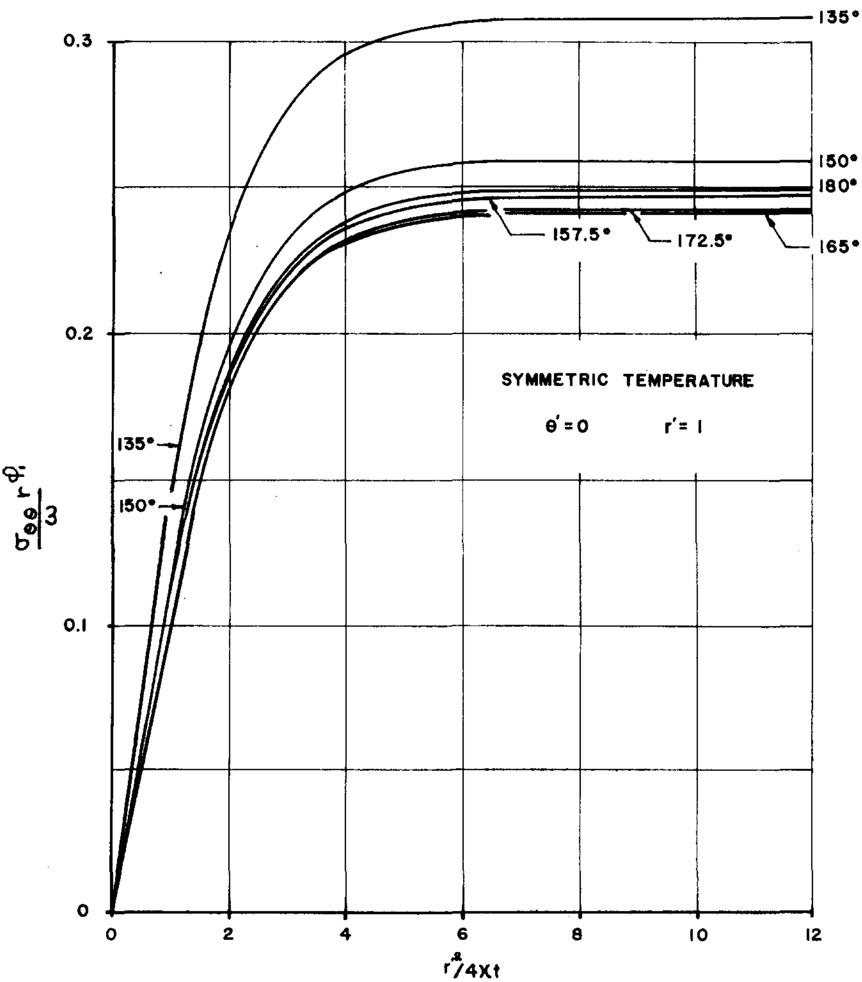

FIG. 7. Time dependence of $\sigma_{\theta\theta}$ on the axis of symmetry with $r < 1$ for various β.

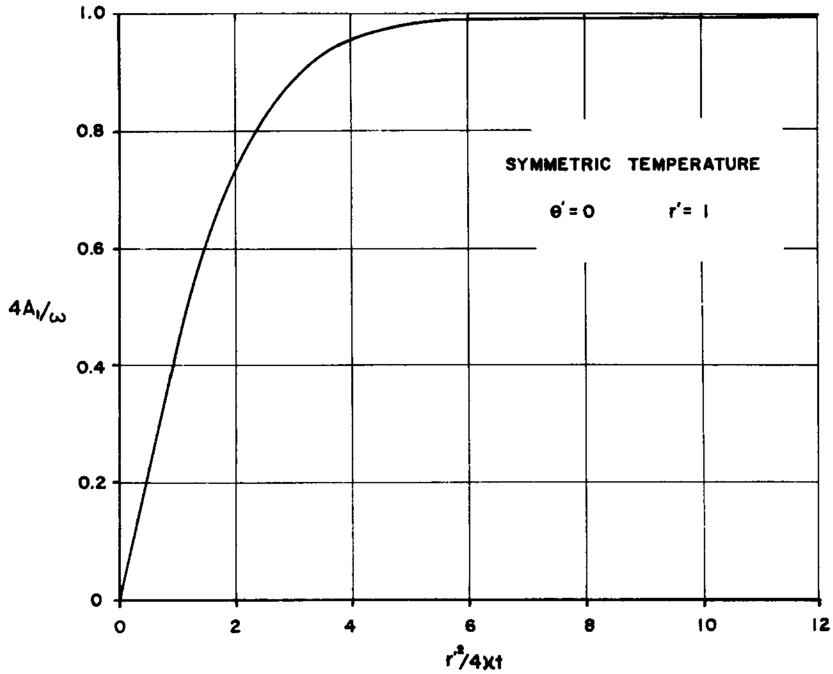

FIG. 8. Time dependence of the stress intensity factor near the apex of a wedge ($\beta = 180°$)—symmetric temperature distribution.

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Résumé—Le problème thermoélastique transitoire sur surface plane d'une forme triangulaire infinie, soumise à une source de chaleur instantanée, en un endroit arbitraire, y est considéré. Le problème est formulé en termes de déplacements et les transformations de Mellin–Laplace sont employées pour la solution. Une représentation intégrale réelle du champs d'effort est donnée. Une insistance particulière est donnée à l'analyse du comportement singulier des efforts dans la région du sommet pour des grandes valeurs de l'angle 2β. L'on a constaté que pour $\beta > \pi/2$ dans le cas symétrique, et que pour $\beta > 0,715\pi$ dans le cas anti-symétrique, le sommet est un point de singularité pour les efforts. La puissance de cette singularité ainsi que la variation des efforts dans θ ainsi que la variation du facteur d'intensité d'effort (temps) est étudié, et quelques résultats numériques y sont donnés.

Zusammenfassung—Das thermoelastische momentane Problem für einen unendlichen ebenen Keil, welcher einer momentanen Wärmequelle in einer willkürlichen Auslegung unterworfen wird, ist erwogen. Das Problem ist formuliert in Beziehung zu Verschiebungen und Mellin–Laplace Umformungen werden für die Auflösung verwendet. Eine Realintegral Darstellung des Beanspruchungsbereiches ist gegeben. Der hauptsächlichste Nachdruck ist ant die Analyse der Singularitäten der Beanspruchungen rings um die Scheitel für grosse Werte des Keilwinkels, 2β, gestellt. Es wird gefunden, das im symmetrischen Fall für $\beta > \pi/2$ und im antisymmetrischen Fall für $\beta > 0,715\pi$ der Scheitel ist ein Punkteiner Singularität für die Beanspruchungen. Die dies Singularität wie auch die Abhängigkeit der Beanspruchungen von θ und die des Beanspruchungs Intensitätsfaktors in Zeit wurden untersucht und einige zahlenmässige Ergebnisse sind gegeben.

Абстракт—Обсуждается простая переходная термоэластическая проблема для бесконечного клина, подвергнутого мгновенному источнику тепла в произвольном месте. Проблема формулируется в условиях перемещения и для решения применяются превращения Меллин–Лапласа (MELLIN–LAPLACE). Дано настоящее полное представление области напряжения. Главное значение придаётся анализу своеобразного поведения напряжений вокруг верхушки для больших значений угла клина 2β. Найдено, что в случае симметрии для $\beta > \pi/2$ и в случае ассиметрии для $\beta > 0.715\ \pi$ верхушка представляет сингулярную точку для напряжений. Изучается сила этой сингулярности, также, как и разнообразие напряжений в β, фактор интенсивности напряжения во времени и даны некоторые числовые результаты.